Algebra II

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Notes
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12-6-13 8-1 Multiplying and Dividing Rational Expressions

How to factor a Quadratic Reminder:

Factoring: $x^2 + 7x + 12$

$a=1 \quad b=7 \quad c=12$

$(x+3)(x+4)$

c	a x b
12	7
3.4	3+4=7

Conjugate pair =

$(x+3)(x-3)$

$x^2 - 3x + 3x - 9$

Cancels

$x^2 - 9 \leftarrow (x+3)(x-3)$

Difference of squares (perfect squares)

-9 | 0
3, -3

$x^2 - 1 \rightarrow (x+1)(x-1)$

Factor Differences of Squares:

A) Factor $m^2 - 64$

$(m+8)(m-8)$

B) $16y^2 - 81z^2$

$(4y+9z)(4y-9z)$

C) $3b^3 - 27b$ ← (factor out common factor)

$3b(b^2-9) \rightarrow 3b(b+3)(b-3)$

1. $b^2 - 9$

$(b+3)(b-3)$

2. $25a^2 - 36b^2$

$(5a+6b)(5a-6b)$

3. $5x^3 - 20x$

$5x(x^2+4)(x^2-4)$

Simplify — when is it undefined?

1. $\dfrac{3y(y+7)}{(y+7)(y^2-9)} = \dfrac{3y(y+7)}{(y+7)(y+3)(y-3)}$

denominator can't be $\emptyset$

$\dfrac{3y\cancel{(y+7)}}{\cancel{(y+7)}(y+3)(y-3)}$

$\dfrac{3y}{(y+3)(y-3)}$

$y \neq -7, -3, 3$

undefined:

$(y+7)(y+3)(y-3) = 0$

$y = -7 \quad y = -3 \quad y = 3$

$y \neq -7, -3, 3$

must be part of answer

2. Simplify:

-3	2
3 · -1	

-15	-2
-5 · 3	

$$\frac{p^2+2p-3}{p^2-2p-15} = \frac{1(p+3)(p-1)}{(p-5)(p+3)1}$$

← must factor first

$$\boxed{\frac{(p-1)}{(p-5)} \quad p \neq 5, -3}$$

undefined:

$(p-5)(p+3)=0$

$p=5 \quad p=-3$

$p \neq 5, -3$

Simplify:

3. $$\frac{a^4b-2a^4}{2a^3-a^3b} = \frac{a^4(b-2)}{a^3(2-b)} =$$

$$\frac{a(b-2)}{(2-b)} = \frac{a(b-2)}{(-b+2)}$$

factor out -1 only when everything is opposite

$$\frac{a(b-2)}{-1(b-2)}$$

$$\frac{a}{-1} = -a$$

$a \neq 0$
$b \neq 2$

undefined:

$a^3(2-b)=0$

$a=0 \quad 2-b=0$

$b=2$

$-2 \quad -2$

$-b=-2$

$b=2$

$$\boxed{-a \quad \begin{matrix} a \neq 0 \\ b \neq 2 \end{matrix}}$$

4. $\dfrac{x(x+5)}{(x+5)(x^2-16)} = \dfrac{x(x+5)}{(x+5)(x+4)(x-4)}$

$\dfrac{x}{(x+4)(x-4)}$

$x \neq -5, -4, 4$

undefined:

$(x+5)(x+4)(x-4) = 0$

$x = -5 \quad x = -4 \quad x = 4$

$x \neq -5, -4, 4$

6	5
2·3	2+3

15	8
3·5	3+5

5. $\dfrac{p^2+5p+6}{p^2+8p+15} = \dfrac{(p+2)(p+3)}{(p+3)(p+5)}$

$\dfrac{(p+2)}{(p+5)} \quad p \neq -3, -5$

undefined:

$(p+3)(p+5) = 0$

$p = -3 \quad p = -5$

$p \neq -3, -5$

6. $\dfrac{x^4y - 3x^4}{3x^3 - x^3y} = \dfrac{x^4(y-3)}{x^3(3-y)} = \dfrac{x(y-3)}{(3-y)} =$

$\dfrac{x(y-3)}{(-y+3)} = \dfrac{x(y-3)}{-(y-3)} = \dfrac{x}{-1} = -x$

$y \neq 3$

$x \neq 0$

(3)

Same

$$\frac{x(x+5)}{(x+5)(x^2-16)} = \frac{x}{x+5} \cdot \frac{x+5}{x^2-16}$$

multiplying Rational Exponents:
(Just like fractions, multiply straight across)

$$\frac{\cancel{8}^{1}x^{1}}{\cancel{21}^{3}y^{3}} \cdot \frac{\cancel{7}^{1}y^{2}}{\cancel{16}^{2}x^{3}} \quad \leftarrow \text{Simplify First}$$

$$\downarrow$$

$$\frac{1 \cdot 1 \cdot 1 \cdot 1}{3 \cdot 2 \cdot y \cdot x^2} = \boxed{\frac{1}{6x^2y}}$$

Divide Rational Exponents:
(multiply by reciprocal)

* KFC – Keep 1st
Flip second
Change the sign to multiplication

$$\frac{10mk^2}{3c^2d} \div \frac{5m^5}{6c^2d^2}$$

$$\downarrow$$

$$\frac{\cancel{10}^{2}mk^2}{\cancel{3}_{1}c^2\cancel{d}_{1}} \cdot \frac{\cancel{6}^{2}c^2d^{\cancel{2}}}{\cancel{5}_{1}m^{\cancel{5}4}} = \frac{2 \cdot 2 \cdot k^2d}{1 \cdot 1 \cdot 1 \cdot 1 \cdot m^4} = \boxed{\frac{4k^2d}{m^4}}$$

$$\boxed{c \neq 0 \quad d \neq 0 \quad m \neq 0}$$

Divide and simplify:

12. $\dfrac{\dfrac{x^2}{9x^2-4y^2} \cdot \dfrac{2y-3x}{x^3}}{\dfrac{x^3}{2y-3x}}$ ← Flip

$$\frac{x^2}{(3x-2y)(3x+2y)} \cdot \frac{2y-3x}{x^3} = \frac{x^2}{(3x-2y)(3x+2y)} \cdot \frac{-1(3x-2y)}{x^3}$$

$$\frac{-1}{x(3x+2y)}$$

$2y-3x = -3x+2y$

$= -1(3x-2y)$

undefined:

$3x+2y=0$

$-2y \quad -2y$

$\frac{3x}{3} = \frac{-2y}{3}$

$x = -\frac{2}{3}y$

$x \neq \frac{2y}{3}, -\frac{2y}{3}, 0$

$y \neq \frac{3}{2}x, -\frac{3}{2}x, 0$

$3x-2y=0$

$-3x \quad -3x$

$\frac{-2y}{-2} = \frac{-3}{-2}x$

$y = \frac{3}{2}x$

$3x+2y=0$

$-3x \quad -3x$

$\frac{2y}{2} = \frac{-3x}{2}$

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Divide and simplify:

11. $\dfrac{2d+6}{d^2+d-2} \div \dfrac{d+3}{d^2+3d+2}$

$\dfrac{2d+6}{d^2+d-2} \cdot \dfrac{d^2+3d+2}{d+3}$

-2	1
$2\cdot -1$	$2-1$

2	3
$1\cdot 2$	$1+2$

$\dfrac{2(d+3)}{1(d+2)(d-1)} \cdot \dfrac{(d+1)(d+2)}{1(d+3)}$

$= \boxed{\dfrac{2(d+1)}{(d-1)} \quad d \neq -2, -3, 1}$

undefined:

$(d+2)(d-1) = 0$

$d = -2 \quad d = 1$

$(d+3) = 0$

$d = -3$

$d \neq -2, 1, -3$