Alg. II
6-2
Notes
1-10-14

Algebra II - Notes

1-10-14 6-2 Inverse Functions and Relations

PEMDAS — (PE)(MD)(AS) inverses of each other

exponent — radical $\sqrt{\ }$

They undo each other

$\sqrt{\ } \rightarrow \sqrt[2]{\ }$

What is the inverse of:

$+$	$-$
$\times$	$\div$
x^2	$\sqrt{\ }$
x^3	$\sqrt[3]{\ }$

* Inverse of a relation/function:

You get the inverse of a relation by exchanging the x- and y-coordinates of all points or exchanging the x- and y-variables in an equation.

Notation → $f^{-1}(x)$

The inverse of $f(x)$ "f of x"

is $f^{-1}(x)$ "f inverse of x"

or "the inverse of f of x"

Ex: $f(x) = \{(-2,-3), (0,-1), (2,2), (4,6)\}$

List the points known to be the inverse of

$f(x) \rightarrow f^{-1}(x) = \{(-3,-2), (-1,0), (2,2), (6,4)\}$

Ex: $f(x) = \{(5,7), (-12,2), (-3,-3), (1,4)\}$

Inverse of $f(x)$ =

$f^{-1}(x) = \{(7,5), (2,-12), (-3,-3), (4,1)\}$

These are inverse functions:

$f(x) = x+3$ (adds 3 on . . .) $g(x) = x-3$ (takes 3 off . . .)

$f(7) = 7+3 = 10$ $g(10) = 10-3 = 7$

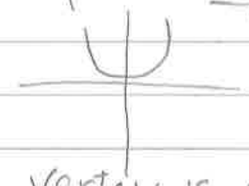
* How to build an inverse:

Given: $f(x) = (x-2)^2 + 5$ ← (parabola - quadratic)

Vertex - (2, 5)

opens up

Step 1: Change from function notation → $f(x) = y$



Step 2: switch the x- and y-

$x = (y-2)^2 + 5$

Vertex is a min.

* Can make a table

x	y
0	9
1	6

Step 3: Solve for y

$x = (y-2)^2 + 5$

(Reverse PERMDAS — parentheses go last)

might have to do PERMDAS more than once — with a fraction → fraction bar is like a parentheses)

$x = (y-2)^2 + 5$
$-5 \qquad\qquad -5$

$\sqrt{x-5} = \sqrt{(y-2)^2}$

$\pm\sqrt{x-5} = y - 2$
$+2 \qquad\quad +2$

$= 2 \pm \sqrt{x-5} = y$

If you $\sqrt{\ }$ remember ±

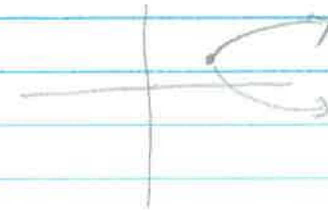
(only when you create $\sqrt{\ }$)

Step 4: Change back to function notation:

$f^{-1}(x) = 2 \pm \sqrt{x-5}$ ← square root family

vertex (5, 2)

2 graphs − and +



Ex: Given $f(x) = \sqrt{x+4} - 3$

1. $y = \sqrt{x+4} - 3$
2. $x = \sqrt{y+4} - 3$ vertex - (−4, −3)
3. $x = \sqrt{y+4} - 3$

$\quad +3 \qquad\qquad +3$

$(x+3)^2 = \sqrt{y+4}^{\,2}$

$(x+3)^2 = y + 4$

$\quad -4 \qquad\quad -4$

$(x+3)^2 - 4 = y$

4. $f^{-1}(x) = (x+3)^2 - 4$ vertex (−3, −4)

Practice:

(1) $f(x) = 4x - 7$

1. $y = 4x - 7$
2. $x = 4y - 7$
3. Solve = $x = 4y - 7$

$$x + 7 = 4y$$

$$\frac{x+7}{4} = \frac{4y}{4}$$

$$\frac{x+7}{4} = y$$

4. $f^{-1}(x) = \frac{x+7}{4}$

(2) $h(x) = \frac{-3x+2}{6}$

1. $y = \frac{-3x+2}{6}$
2. $x = \frac{-3y+2}{6}$
3. $(6)x = \frac{-3y+2}{6}(6)$

$$6x = -3y + 2$$

$$-2 \qquad -2$$

$$\frac{6x-2}{-3} = \frac{-3y}{-3}$$

4. $f^{-1}(x) = \frac{6x-2}{-3}$

(3) $g(x) = 4 + (2x-3)^2$

1. $y = 4 + (2x-3)^2$

2. $x = 4 + (2y-3)^2$

3. $x = 4 + (2y-3)^2$

$-4 \quad -4$

$\pm\sqrt{x-4} = \sqrt{(2y-3)^2}$

$\pm\sqrt{x-4} = 2y-3$

$+3 \quad +3$

$\frac{3 \pm \sqrt{x-4}}{2} = \frac{2y}{2}$

$\frac{3 \pm \sqrt{x-4}}{2} = y$

4. $g^{-1}(x) = \frac{3 \pm \sqrt{x-4}}{2}$

(4) $f(x) = 3 + \sqrt{2(x+4)}$

1. $y = 3 + \sqrt{2(x+4)}$

2. $x = 3 + \sqrt{2(y+4)}$

3. $x = 3 + \sqrt{2(y+4)}$

$\quad -3 \quad\ -3$

$(x-3)^2 = \sqrt{2(y+4)}^{\,2}$

$\frac{(x-3)^2}{2} = \frac{2(y+4)}{2}$

$\frac{(x-3)^2}{2} = y + 4$

$\quad -4 \quad -4$

$\frac{(x-3)^2}{2} - 4 = y$

4. $f^{-1}(x) = \frac{(x-3)^2}{2} - 4$