Algebra II - Notes

Alg II
5-1
Notes
11-12-13

11-12-13 5-1 Operations with Polynomials

monomial: a polynomial with one term (only multiplication)

Ex: 3, $3x+3x+(6x)$, $3x$ or $3x^2y^3z$ or $\frac{1}{2}xmq$ or $-\frac{3}{4}m^2n$

binomial: a polynomial with 2 terms (adding or subtracting)

Ex: $3+3x$, $3-3x$, $3x^2y^3z - \frac{3}{4}m^2n$

trinomial: a polynomial with 3 terms

Ex: $3x^2+2x+5$, $3+3x+3x^2y^3z$

A monomial expression is in simplified form when:

- There are no powers of a power — Not simplified: $(x^3)^4$, $\times 4$
- Each base appears exactly once
- All fractions are in simplest form $\rightarrow \frac{10}{4} = \frac{5}{2}$
- There are no negative exponents

multiply and Divide monomials:
AKA: Exponent properties

1. **Product Property** $a^m a^n = a^{m+n}$
   Ex: $a^2 a^3 = aa\,aaa = a^5$

Ex: $K^4K^2 = K^{4+2} = K^6$

$4m^2(3m^7) = 12m^9$

$\downarrow$

$4 \cdot 3 \cdot m^2 \cdot m^7$

① $(2K^4)(4K^2) = \boxed{8K^6}$

② $(j^2K^4)(j^3K^{-1}) = \boxed{j^5K^3}$

③ $-3x^2y^3(4xy^3)2y = -3\cdot4\cdot2\cdot x^2\cdot x\cdot y^3y^3y = \boxed{-24x^3y^7}$

2. Quotient Property: $\frac{a^n}{a^m} = a^{n-m}$

Ex: $\frac{a^6}{a^4} = \frac{\cancel{a}\cancel{a}\cancel{a}\cancel{a}aa}{\cancel{a}\cancel{a}\cancel{a}\cancel{a}} = \frac{a^2}{1} = a^2$

$\frac{a^7}{a^3} = a^{7-3} = a^4$

$\frac{1a^4}{a^{\cancel{7}3}} = \frac{1}{a^3}$ or $a^{4-7} = a^{-3} = \frac{1}{a^3}$

$\frac{\cancel{a}\cancel{a}\cancel{a}\cancel{a}}{\cancel{a}\cancel{a}\cancel{a}\cancel{a}aaa} = \frac{1}{a^3}$

practice: a) $\frac{2k^9}{6k^7} = \frac{1}{3}k^{9-7} = \boxed{\frac{1}{3}k^2}$

b) $\frac{k^7}{k^3} = k^{7-3} = \boxed{k^4}$

c) $\frac{j^2k^4}{6j^5k^7} = \boxed{\frac{1}{6j^3k^3}}$

3. Negative Exponents $= \frac{1}{a^{-1}} = a^1$ and

$\frac{1}{a} = a^{-1}$

Ex: $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

$\frac{2x^3y^{-2}}{4x^{-5}y} = \frac{2x^3x^5}{4y^2y} = \boxed{\frac{1x^8}{2y^3}}$

$2(a^{-7}) = \boxed{\frac{2}{a^7}}$

$-5x^{-3} = \boxed{\frac{-5}{x^3}}$

a) $2x^{-6} = \boxed{\dfrac{2}{x^6}}$

b) $\dfrac{k^{-3}}{k} = \dfrac{1}{k^3 k} = \boxed{\dfrac{1}{k^4}}$

c) $\dfrac{1}{k^{-2}} = 1k^2 = \boxed{k^2}$

d) $(3xyz^7)^{-1} = \dfrac{1}{3xyz^7}$

e) $\dfrac{4k^{-1}m^3n^{-2}}{8km^{-4}n^5} = \dfrac{4m^3m^4}{8k^1n^2n^5} = \dfrac{1m^7}{2kn^7} = \boxed{\dfrac{m^7}{2kn^7}}$

f) $(3xyz^7)^{-7} = \dfrac{1}{(3xyz^7)^7}$

9. Zero Exponents = $a^0 = 1$

Anything to the zero power will be one!

$\dfrac{x^5}{x^5} = 1 \rightarrow x^{5-5} = x^0 = 1$

Ex: $4x^0 = 4 \cdot 1 = 4$

$(4x)^0 = 1$

a) $k^0 = 1$

b) $(2k^{-4})^0 = 1$

c) $\left(\frac{1}{j^3 k^{-2}}\right)^0 = 1$

d) $2x^0 = 2 \cdot 1 = 2$

e) $(2x)^0 = 1$

Steps
- Circle negative exponents → move it
- Circle zero exponents → substitute 1
- Expand / simplify
  apply rules

a) $(x^3)^4 = x^3 x^3 x^3 x^3 = \boxed{x^{12}}$

b) $(x^{-2})^6 = \left(\frac{1}{x^2}\right)^6 = \frac{1}{x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} = \boxed{\frac{1}{x^{12}}}$

c) $(k^{x-2})^3 = k^{x-2}\, k^{x-2}\, k^{x-2} = k^{3x-6}$

d) $(x^m)^4 = x^m \cdot x^m \cdot x^m \cdot x^m = x^{m+m+m+m} = \boxed{x^{4m}}$

a) $(rd)^2 = rd \cdot rd = r^2 d^2$

b) $(3k^4)^3 = 3k^4 \cdot 3k^4 \cdot 3k^4 = 27k^{12} = 3^3 k^{12}$

c) $(4j^{-2}k^4)^5 = \left(\frac{4^1 k^4}{j^2}\right)^5 = \frac{4k^4}{j^2} \cdot \frac{4k^4}{j^2} \cdot \frac{4k^4}{j^2} \cdot \frac{4k^4}{j^2} \cdot \frac{4k^4}{j^2} = \frac{4^5 k^{20}}{j^{10}}$

d) $(4^2 x^{-6} y^3 z^0)^3 = \left(\frac{16y^3}{x^6}\right)^3 = \frac{16^3 y^9}{x^{18}}$

$\frac{16y^3}{x^6} \cdot \frac{16y^3}{x^6} \cdot \frac{16y^3}{x^6} = \frac{4096\, y^9}{x^{18}}$

e) $\left(\frac{x}{y}\right)^3 = \frac{x^1}{y^1} \cdot \frac{x^1}{y^1} \cdot \frac{x^1}{y^1} = \frac{x^3}{y^3}$

f) $\left(\frac{k^4}{k^2}\right)^3 = (k^2)^3 = k^2 \cdot k^2 \cdot k^2 = k^6$

g) $\left(\frac{4j^2k^4}{2j^3k^{-1}}\right)^2 = \left(\frac{4j^2k^4k^1}{2j^3}\right)^2 = \left(\frac{2k^5}{j}\right)^2 =$

$\frac{2k^5}{j} \cdot \frac{2k^5}{j} = \frac{4k^{10}}{j^2}$