Algebra 2 T7-1 RETAKE WS Name Key

Target 7-1: I can describe transformations, domain and range, and graph exponential and logarithmic functions.

1. $y = -4\left(\frac{1}{2}\right)^{x+2}$

Parent function: $y = \left(\frac{1}{2}\right)^x$

$a = -4$ meaning v. flip, stretch by 4

$h = -2$ meaning left 2

$k = 0$ meaning n/a

Domain: $\mathbb{R}$

Range: $y < 0$

2. $y = \frac{1}{5}(2)^{x-1}$

Parent function: $y = (2)^x$

$a = \frac{1}{5}$ meaning v shrink by 5

$h = 1$ meaning right 1

$k = 0$ meaning n/a

Domain: $\mathbb{R}$

Range: $y > 0$

3. $y = 2\left(\frac{1}{5}\right)^x - 1$

Parent function: $y = \left(\frac{1}{5}\right)^x$

$a = 2$ meaning v st by 2

$h = 0$ meaning n/a

$k = -1$ meaning down 1

Domain: $\mathbb{R}$

Range: $y > -1$

4. $y = \frac{1}{2}(3)^{x+4} - 5$

Parent function: $y = 3^x$

$a = \frac{1}{2}$ meaning v shrink by 2

$h = -4$ meaning left 4

$k = -5$ meaning down 5

Domain: $\mathbb{R}$

Range: $y > -5$

5. $y = 3\log_2 x + 1$

Parent function: $y = \log_2 x \Rightarrow 2^y = x$

$a = 3$ meaning v st by 3

$h = 0$ meaning n/a

$k = 1$ meaning up 1

Domain: $x > 0$

Range: $\mathbb{R}$

6. $y = 2\log_{\frac{1}{4}}(x-3) + 2$

Parent function: $y = \log_{\frac{1}{4}} x \Rightarrow \left(\frac{1}{4}\right)^y = x$

$a = 2$ meaning v st by 2

$h = 3$ meaning right 3

$k = 2$ meaning up 2

Domain: $x > 3$

Range: $\mathbb{R}$

7. $y = \frac{1}{2}\log_8(x+1) - 2$

Parent function: $y = \log_8 x \Rightarrow 8^y = x$

$a = \frac{1}{2}$ meaning v shrink by 2

$h = -1$ meaning left 1

$k = -2$ meaning down 2

Domain: $x > -1$

Range: $\mathbb{R}$

8. $y = -2\log_2(x-4) + 3$

Parent function: $y = \log_2 x$

$a = -2$ meaning v flip v st by 2

$h = 4$ meaning right 4

$k = 3$ meaning up 3

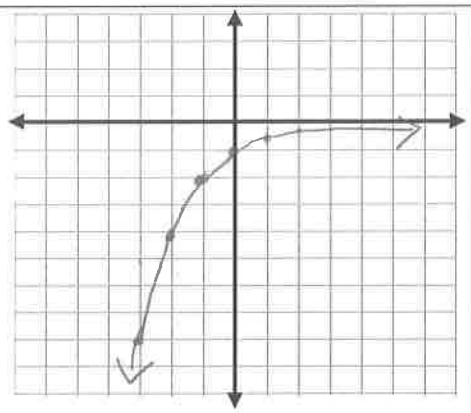
Domain: $x > 4$

Range: $\mathbb{R}$

Graphs for T7-1

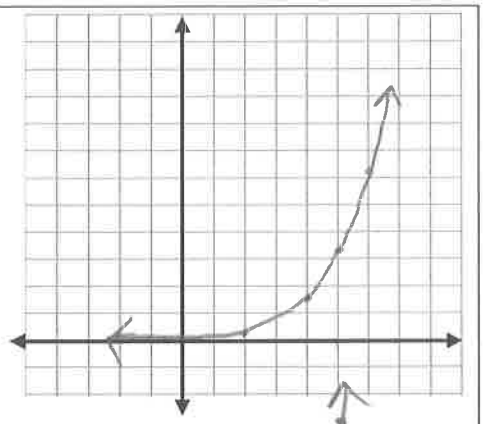
1. $y = -4\left(\frac{1}{2}\right)^{x+2}$

x	y
-4	-16
-3	-8
-2	-4
-1	-2
0	-1
1	-.5
2	-.25



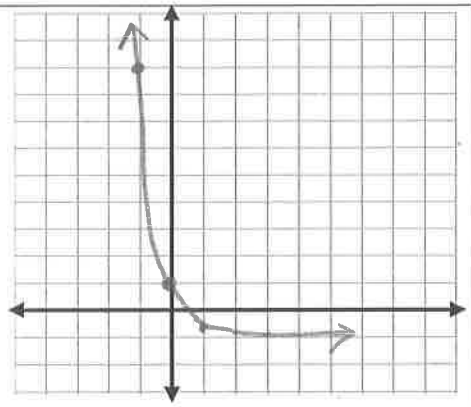
2. $y = \frac{1}{5}(2)^{x-1}$

-2	.025
-1	.05
0	.1
1	.2
2	.4
4	1.6
5	3.2



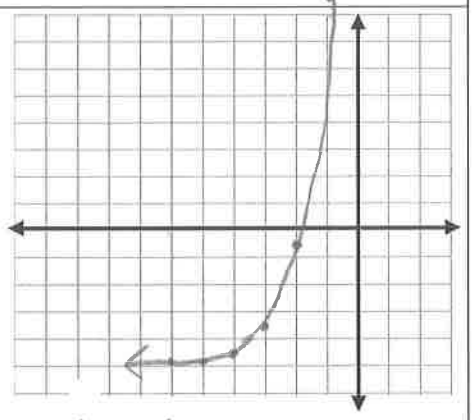
3. $y = 2\left(\frac{1}{3}\right)^{x} - 1$

x	y
-2	49
-1	9
0	1
1	-.6
2	-.92
3	-.99



4. $y = \frac{1}{2}(3)^{x+4} - 5$

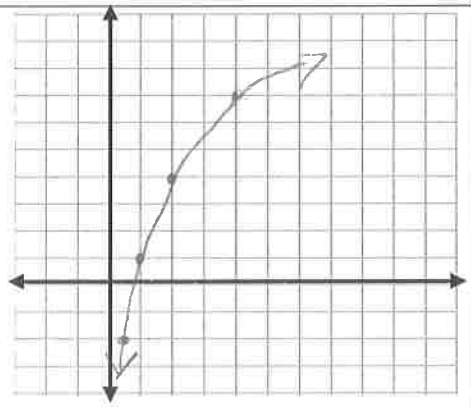
-6	-4.9
-5	-4.8
-4	-4.5
-3	-3.5
-2	-.5
-1	8.5
0	35.5



5. $y = 2^{x}$ $\quad x = 2^{y}$

-1	1/2	1/2	-1
0	1	1	0
1	2	2	1
2	4	4	2

x	y
1/2	-2
1	1
2	4
4	7

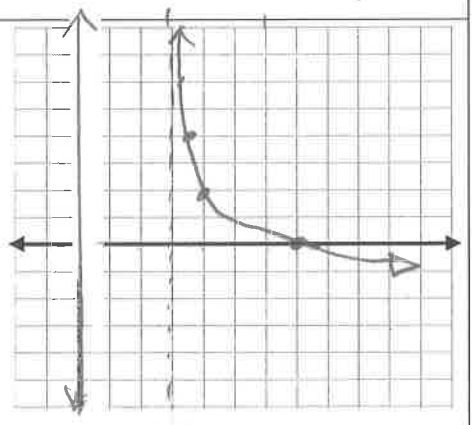


6. $y = \left(\frac{1}{4}\right)^{x}$ $\quad x = \left(\frac{1}{4}\right)^{y}$

-1	4	4	-1
0	1	1	0
1	1/4	1/4	1
2	1/16	1/16	2

$\log_{4} 2 + 2$

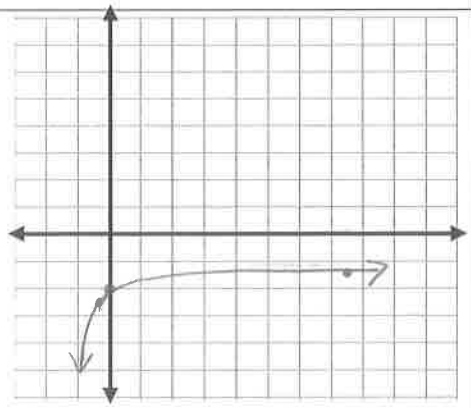
7	0
4	2
3.25	4
3.…	6



7. $y = 8^{x}$

-1	1/8	1/8	-1
0	1	1	0
1	8	8	1
2	64	64	2

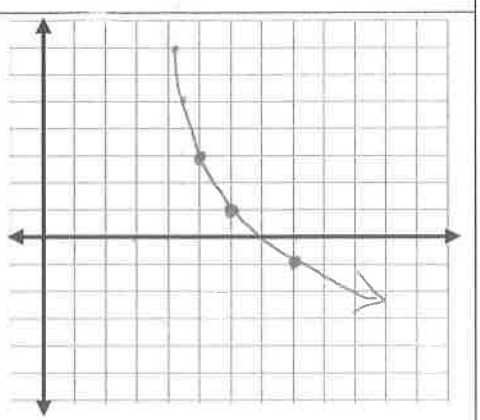
x	y
-.875	-2.5
0	-2
7	-1.5
63	-1



8. $y = 2^{x}$ $\quad 2^{y} = x$

-2	1/4	1/4	-2
-1	1/2	1/2	-1
0	1	1	0
1	2	2	1
2	4	4	2

4.25	7
4.5	5
5	3
6	1
8	-1



NAME Key DATE _____ PERIOD _____

T7-2 RETAKE WS

Solving & Writing Exponential Equations

Solve each equation.

1. $25^{x+5} = 125^{x-3}$

$(5^2)^{x+5} \quad (5^3)^{x-3}$

$2(x+5) = 3(x-3)$

$2x+10 = 3x-9$

$19 = x$ ✓

2. $\left(\frac{1}{27}\right)^{\frac{1}{3}x-5} = 9^{x-3}$

$(3^{-3})^{\frac{1}{3}x-5} = (3^2)^{x-7}$

$-3\left(\frac{1}{3}x-5\right) = 2(x-3)$

$-x+15 = 2x-6$

$+x \quad +6 \quad +x \quad +6$

$\frac{21}{3} = \frac{3x}{3}$

$x = 7$

3. $11^{x-4} = 121^{x+28}$

$(11^2)^{x+28}$

$x-4 = 2(x+28)$

$x-4 = 2x+56$

$x = -60$

4. $\left(\frac{1}{6}\right)^{2x+2} = 216^{x-1}$

$(6^{-1})^{2x+2} = (6^3)^{x-1}$

$-1(2x+2) = 3(x-1)$

$-2x-2 = 3x-3$

$\frac{1}{5} = \frac{5x}{5}$

$x = \frac{1}{5}$

5. $\left(\frac{1}{2}\right)^{x-3} = 16^{3x+1}$

$x = -\frac{1}{13}$

6. $3^{6x-2} = \left(\frac{1}{9}\right)^{x+1}$

$x = 0$

Write an exponential function in the form $y = ab^x$ for the graph that passes through the given points.

7. (0, 5) and (4, 3125)

$3125 = 5(b)^4$

$y = 5(5)^x$

8. (0, 8) and (4, 2048)

$y = 8(4)^x$

9. $\left(0, \frac{3}{4}\right)$ and (2, 36.75)

$y = 0.75(7)^x$

10. (0, –0.2) and (–3, –3.125)

$y = -0.2(0.4)^x$

11. (0, 15) and $\left(2, \frac{15}{16}\right)$

$y = 15\left(\frac{1}{4}\right)^x$

12. (0, 0.7) and $\left(\frac{1}{2}, 3.5\right)$

$y = 0.7(25)^x$

Solve each equation.

13. $20 \cdot 400 = \left(\frac{1}{20}\right)^{7x+11}$

$20^1 \cdot 20^2 = (20^{-1})^{7x+11}$

$20^3 = 20^{-7x-11}$

$+11 \quad 3 = -7x-11 \quad +11$

$14 = -7x$

$x = -2$

14. $10^{4x+8} = 1000^x$

$4x+8 = 3x$

$x = -8$

15. $\left(\frac{1}{16}\right)^{3x-4} = 64^{x-1}$

$x = \frac{11}{9}$

16. $\left(\frac{1}{8}\right)^{x-6} = 4^{4x+5}$

$x = \frac{8}{11}$

17. $\left(\frac{1}{36}\right)^{x+8} = 6^x \cdot 216^{x-3}$

$(6^2)^{x+8} = 6^x \, 6^{3(x-3)}$

$-2(x+8) = x+3(x-3)$

$-2x-16 = x+3x-9$

$\frac{6x}{6} = \frac{-7}{6}$

$x = -\frac{7}{6}$

18. $128^{x+3} = \left(\frac{1}{1024}\right)^{2x}$

$(2^7)^{x+3} = (2^{-10})^{2x}$

$7(x+3) = -10(2x)$

$7x+21 = -20x$

$\frac{21}{27} = \frac{-27x}{-27}$

$x = \frac{-21}{27} = -\frac{7}{9}$

19. At time t, there are 216^{t+18} bacteria of type A and 36^{2t+8} bacteria of type B organisms in a sample. When will the number of each type of bacteria be equal?

$216^{t+18} = 36^{2t+8}$

$(6^3)^{t+18} = (6^2)^{2t+8}$

$3(t+18) = 2(2t+8)$

$3t+54 = 4t+16$

$38 = t$

When $t = 38$ (no units given)

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T7-3 RETAKE WS

Side 1 – w/o properties

Solving Logarithmic Equations

Solve each equation.

1. $3x = \log_6 216$

$x=1$

2. $x - 4 = \log_3 243$

$x = 9$

3. $\log_4 (4x - 20) = 5$

$x=261$

4. $\log_9 (3 - x) = \log_9 (5x - 15)$

no sdution

5. $\log_{81} (x + 20) = \log_{81} (6x)$

$x=4$

6. $\log_9 (3x^2) = \log_9 (2x + 1)$

$x=-\frac{1}{3}$ or $x=1$

7. $\log_4 (x - 1) = \log_4 (12)$

$x=13$

8. $\log_7 (5 - x) = \log_7 (5)$

$x=0$

9. $\log_x (5x) = 2$

$x=5$

Solve each equation.

10. $\log_5 (-3x) = 1$

$5^1 = -3x$

$x=-\frac{5}{3}$

11. $\log_6 x = \log_6 (4 - x)$

$x = 4-x$

$2x=4$

$x=\frac{1}{2}$

12. $\log_{10} (x - 3) = 2$

$100 = x-3$

$x=103$

13. $\log_2 (x - 5) = \log_2 (3)$

$x-5=3$

$x=8$

14. $\log_7 (8x + 5) = \log_7 (6x - 18)$

$8x+5=6x-18$

$2x=-23$

no solution

15. $\log_9 (3x - 3) = 1.5$

$9^{1.5} = 3x-3$

$x=10$

16. $\log_{10} (2x - 2) = \log_{10} (7 - x)$

$2x-2=7-x$

$3x=9$

$x=3$

17. $\log_9 (x - 1) = \log_9 (2x)$

$x-1=2x$

$-1=x$

18. $\log_{16} x = 0.5$ ~~$\frac{1}{2}$~~

$16^{.5} = x$

$x=4$

19. $\log_3 \left(\frac{x-3}{4} + 5\right) = \log_3 (x + 2)$

$\frac{x-3}{4} + 5 = x+2$

$x-3=4x-12$

$9=3x$

$x=3$

20. $\log_5 (3x) = \log_5 (2x - 1)$

$3x=2x-1$

$x=-1$

21. $\log_3 (7 - x) = \log_3 (x + 19)$

$7-x=x+19$

$-12=2x$

$x=-6$

NAME key DATE _____ PERIOD _____

T7-3 RETAKE WS

Side 2 – w/ properties

Solving Logarithmic Equations

Solve each equation. Check your solutions.

1. $\log_7 n = \frac{2}{3}\log_7 8$ $n=4$

2. $\log_{10} u = \frac{3}{2}\log_{10} 4$ $u=8$

3. $\log_6 x + \log_6 5 = \log_6 45$ $x=9$

4. $\log_8 32 - \log_8 w = \log_8 4$ $w=8$

5. $\log_9 (3u + 14) - \log_9 5 = \log_9 2u$ $u=2$

6. $4\log_2 x + \log_2 5 = \log_2 405$ $x=3$

7. $\log_3 y = -\log_3 16 + \frac{1}{3}\log_3 64$ $y=\frac{1}{4}$

8. $\log_2 d = 5\log_2 2 - \log_2 8$ $d=4$

9. $\log_{10} (3m - 5) + \log_{10} m = \log_{10} 2$ $m=2$

10. $\log_{10} (b + 3) + \log_{10} b = \log_{10} 4$ $b=1$

11. $\log_8 (t + 10) - \log_8 (t - 1) = \log_8 12$ $t=2$

12. $\log_3 (a + 3) + \log_3 (a + 2) = \log_3 6$ $a=0$

13. $\log_{10} (r + 4) - \log_{10} r = \log_{10} (r + 1)$ $r=2$

14. $\log_4 (x^2 - 4) - \log_4 (x + 2) = \log_4 1$ $x=3$

15. $\log_{10} 4 + \log_{10} w = 2$ $w=25$

16. $\log_8 (n - 3) + \log_8 (n + 4) = 1$ $n=4$

17. $3\log_5 (x^2 + 9) - 6 = 0$ $x=\pm 4$

18. $\log_{16} (9x + 5) - \log_{16} (x^2 - 1) = \frac{1}{2}$ $x=3$

19. $\log_6 (2x - 5) + 1 = \log_6 (7x + 10)$ $x=8$

20. $\log_2 (5y + 2) - 1 = \log_2 (1 - 2y)$ $y=0$

NAME _____ DATE _____ PERIOD _____

7-5 Practice

Properties of Logarithms

Use $\log_{10} 5 \approx 0.6990$ and $\log_{10} 7 \approx 0.8451$ to approximate the value of each expression.

1. $\log_{10} 35$ **1.5441**
2. $\log_{10} 25$ **1.3980**
3. $\log_{10} \frac{7}{5}$ **0.1461**
4. $\log_{10} \frac{5}{7}$ **−0.1461**
5. $\log_{10} 245$ **2.3892**
6. $\log_{10} 175$ **2.2431**
7. $\log_{10} 0.2$ **−0.6990**
8. $\log_{10} \frac{25}{7}$ **0.5529**

Solve each equation. Check your solutions.

9. $\log_7 n = \frac{2}{3} \log_7 8$ **4**
10. $\log_{10} u = \frac{3}{2} \log_{10} 4$ **8**
11. $\log_6 x + \log_6 9 = \log_6 54$ **6**
12. $\log_8 48 - \log_8 w = \log_8 4$ **12**
13. $\log_9 (3u + 14) - \log_9 5 = \log_9 2u$ **2**
14. $4 \log_2 x + \log_2 5 = \log_2 405$ **3**
15. $\log_3 y = -\log_3 16 + \frac{1}{3} \log_3 64$ $\frac{1}{4}$
16. $\log_2 d = 5 \log_2 2 - \log_2 8$ **4**
17. $\log_{10} (3m - 5) + \log_{10} m = \log_{10} 2$ **2**
18. $\log_{10} (b + 3) + \log_{10} b = \log_{10} 4$ **1**
19. $\log_8 (t + 10) - \log_8 (t - 1) = \log_8 12$ **2**
20. $\log_3 (a + 3) + \log_3 (a + 2) = \log_3 6$ **0**
21. $\log_{10} (r + 4) - \log_{10} r = \log_{10} (r + 1)$ **2**
22. $\log_4 (x^2 - 4) - \log_4 (x + 2) = \log_4 1$ **3**
23. $\log_{10} 4 + \log_{10} w = 2$ **25**
24. $\log_8 (n - 3) + \log_8 (n + 4) = 1$ **4**
25. $3 \log_5 (x^2 + 9) - 6 = 0$ **±4**
26. $\log_{16} (9x + 5) - \log_{16} (x^2 - 1) = \frac{1}{2}$ **3**
27. $\log_6 (2x - 5) + 1 = \log_6 (7x + 10)$ **8**
28. $\log_2 (5y + 2) - 1 = \log_2 (1 - 2y)$ **0**
29. $\log_{10} (c^2 - 1) - 2 = \log_{10} (c + 1)$ **101**
30. $\log_7 x + 2 \log_7 x - \log_7 3 = \log_7 72$ **6**

31. **SOUND** Recall that the loudness L of a sound in decibels is given by $L = 10 \log_{10} R$, where R is the sound's relative intensity. If the intensity of a certain sound is tripled, by how many decibels does the sound increase? **about 4.8 dB**

32. **EARTHQUAKES** An earthquake rated at 3.5 on the Richter scale is felt by many people, and an earthquake rated at 4.5 may cause local damage. The Richter scale magnitude reading m is given by $m = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion. How many times greater is the amplitude of an earthquake that measures 4.5 on the Richter scale than one that measures 3.5? **10 times**

Target 7-4 RETAKE WORKSHEET Name Key Per ____

1. Ten grams of Carbon 14 is stored in a container. The amount C (in grams) of Carbon 14 present after t years can be modeled by $C = 10(0.99987)^t$. How much is present after 1000 years?

$C = 10(0.99987)^{1000}$

≈ 8.18 grams

2. You deposit \$2000 in an account that earns 5% annual interest. Write a function for each of the following frequencies. Then determine the balance after 2 years if the interest is compounded with the given frequency.

a. annually

Function: $y = 2000(1+.05)^x$
Balance: \$2205.00

b. quarterly

Function: $y = 2000(1+\frac{.05}{4})^{4t}$
Balance: \$2208.97

c. monthly

Function: $y = 2000(1+\frac{.05}{12})^{12t}$
Balance: \$2210.17

3. A customer purchases a television for \$800 using a credit card. The interest is charged on an unpaid balance at a rate of 18% per year compounded monthly. If the customer makes no payment for one year, how much is owed at the end of the year?

$y = 800(1+\frac{.18}{12})^{12t}$

\$956.49

4. A diamond ring was purchased twenty years ago for \$500. The value of the ring increases by 8% each year. What is the value of the ring today?

$y = 500(1+.08)^t$ $t = 20$

\$2330.48

5. In 1990 the tuition at a private college was \$15000. During the next 9 years, tuition increased by about 7.2% each year.

a. Write a model giving the cost C of tuition at the college t years after 1990

$C = 15{,}000(1+.072)^t$

b. About what year will the tuition be \$20,000?

$20000 = 15000(1.072)^t$
$1.333 = 1.072^t$
$t \approx 4.1$ So 1994

c. If this trend continues what will the tuition be in 2010?

$t = 20$

\$60,254

6. The number of newly reported cases of tuberculosis in the US in 1991 was 28,500. In 1996 it went down to 22,841. The decrease in cases models exponential decay. Write a function to model this situation where t represents the number of years since 1991.

   a. Identify the initial amount, decay factor and annual percent decrease.
   (1991, 28,500) (1996, 22841) $22841 = 28500b^5$ initial amt 28500
   (0, 28,500) (5, 22841) $b = .9567$ decay factor $b = .956$
   $1 - .9567 = .0433$

   b. In what year was the number of newly reported cases in US approximately 25,000? % Dec = 4.3
   $y = 28500(.9567)^t$ $25000 = 28500(.9567)^t$ $t = 2.96 \approx 3$ years
   $.8772 = .9567^t$ $1991 + 3 =$ **1994**

   c. When will the number of newly reported cases be about 16,000?
   $16000 = 28500(.9567)^t$ $t = 13.04$
   about 2004

   d. Estimate the number of newly reported cases in 2005.
   $t = 14$ about 15335 cases

7. A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

   a. Write an exponential decay model for the value of equipment.
   $y = 250,000(1-.12)^t$ $y = 250,000(.88)^t$

   b. What is the value of equipment after 5 years? About $131,932.98

   c. Approximately when the equipment will have a value of $70,000?
   $t \approx 9.958$ in about 10 years.

8. A house was purchased for $90,000 in 1995. If the value of the home increases 5% per year, what is it worth in the year 2020?

   Function: $y = 90,000(1.05)^t$ $t = 25$
   Worth in 2020: $304,771.94 approx.

9. You deposit $1000 in an account that earns 2.5% annual interest. Find the balance after 3 years if the interest compounds with the given frequency.

   a. **monthly**
   Function: $y = 1000\left(1 + \frac{.025}{12}\right)^{12t}$
   Balance: $1077.80

   b. **daily**
   Function: $y = 1000\left(1 + \frac{.025}{365}\right)^{365t}$
   Balance: $1077.88