Algebra II - Notes

Alg II
T7-2
Notes
2-26-14

2-26-14 T7-2 Solving Exponential Equations and Inequalities

Exponent Rules: $(a^x)^y = a^{xy}$

$a^x \cdot a^y = a^{x+y}$

$a^0 = 1$

$\frac{1}{a^1} = a^{-1}$

* Property of Equality For Exponential Functions:

If $b > 0$ and $b \neq 1$, then
$b^x = b^y$, if and only if $x = y$

If the base is the same, exponent is the same.

Ex: $2^x = 2^2$ so $\rightarrow$ $x = 2$

Ex: $3^{(2x)} = 3^{(x+3)}$ ← bases must be the same

so $\rightarrow$ $2x = x + 3$

$-x \quad -x$

$\boxed{x = 3}$

Verify $\rightarrow$ $3^{2(3)} = 3^{3+3}$

$3^6 = 3^6$ ✓

If the bases are <u>not</u> the same:

1. Get bases the same.
2. Set exponent to equal each other
3. Solve / verify

Ex: $3^x = (9)^4$ $\quad 9 = (3^2)$ ← use substitution

$3^x = (3^2)^4$

$3^x = 3^8$

$\boxed{x = 8}$

Verify:

$3^8 = 9^4$

$6561 = 6561$ ✓

Ex: $2^{5x} = (4)^{2x-1}$ $\quad 4 = (2^2)$

$2^{5x} = (2^2)^{(2x-1)}$

$5x = 2(2x-1)$

$5x = 4x - 2$

$-4x \quad -4x$

$\boxed{x = -2}$

Verify =

$2^{5(-2)} = 4^{2(-2)-1}$

$2^{-10} = 4^{-5}$ ✓

(Done on calculator)

<u>practice</u>: $3^{5x} = 1$ $\quad 1 = 3^0$

$3^{5x} = 3^0$

$\frac{5x}{5} = \frac{0}{5}$

$\boxed{x = 0}$

$3^{(5 \cdot 0)} = 1$

$3^0 = 1$ ✓

②

practice: $(625)^{5x-1} = 25^{6x}$ $(25^2 \neq 625$

$(25^2)^{(5x-1)} = 25^{6x}$

$2(5x-1) = 6x$

$10x - 2 = 6x$

$-6x \quad -6x$

$4x - 2 = 0$

$+2 \quad +2$

$\frac{4x}{4} = \frac{2}{4}$

$\boxed{x = \frac{1}{2}}$

Verify:

$625^{5(\frac{1}{2})-1} = 25^{6(\frac{1}{2})}$

$625^{1.5} = 25^3$ ✓

Keep the base ⟶ $3^{-2x+1} \cdot 3^{-2x-3} = 3^{-x}$

$3^{(-2x+1)+(-2x-3)} = 3^{-x}$

$3^{-4x-2} = 3^{-x}$

$-4x - 2 = -x$

$+4x \quad +4x$

$\frac{-2}{3} = \frac{3x}{3}$

$\boxed{\frac{-2}{3} = x}$

Verify:

$3^{-2(-\frac{2}{3})+1} \cdot 3^{-2(-\frac{2}{3})-3} = 3^{-(-\frac{2}{3})}$

$3^{\frac{4}{3}+1} \cdot 3^{\frac{4}{3}-3} = 3^{\frac{2}{3}}$

5. $4^x = (64)^3$ $\quad 64 = (4^3)$

$4^x = (4^3)^3$

$4^x = 4^9$

$\boxed{x = 9}$

Verify:

$4^9 = 64^3$ ✓

6. $3^{2x} = (9)^{5x-4}$ $\quad 9 = (3^2)$

$3^{2x} = (3^2)^{(5x-4)}$

$2x = 2(5x-4)$

$2x = 10x - 8$

$-2x \quad -2x$

$0 = 8x - 8$

$+8 \quad +8$

$8 = 8x$

$\frac{8}{8} = \frac{8x}{8}$

$\boxed{x = 1}$

Verify:

$3^{2(1)} = 9^{(5(1)-4)}$

$3^2 = 9^1$

$9 = 9$ ✓

7. $216 = \left(\frac{1}{6}\right)^{x+3}$ $\quad 6^3 = 216$

change to (6^{-1})

$6^3 = (6^{-1})^{(x+3)}$

$3 = -1(x+3)$

$3 = -x - 3$

$+3 \quad +3$

$(-1)\ 6 = -x(-1)$

$\boxed{-6 = x}$

$216 = \left(\frac{1}{6}\right)^{-6+3}$

$216 = \left(\frac{1}{6}\right)^{-3}$

$216 = 216$ ✓

Word Problems:

Exponential Growth / Decay:

$y = a(b)^t$ ← time

outcome → y = a (original amt) (b) (growth or decay) ^{t} ← time

$y = a(1 \pm r)^t$ ← time

Final amt. → y = a (initial amount) $(1 \pm r)$ (growth / decay) ^{t} ← time

r = rate of change (converted to a decimal)

$300\% = 3$ $\quad$ $1,237\% = .01237$

When doing exponentials → round to 3 decimal points (unless decimal ends at 4 decimal)

7. Population in 2000 = 1,321,045
2007 ≈ 1,512,986

Write X in terms of the numbers of years since 2000

Write an exponential function to model growth.

(a) (t) (y)

$(0, 1,321,045)$ $\quad$ $(7, 1,512,986)$

starting time → 0 = year 2000; 1,321,045 = population (initial amt); 7 = year 2007 (end time); 1,512,986 = population (end amt.)

Substitute: $1,512,986 = 1,321,045(b)^7$

$$\frac{1,512,986}{1,321,045} = \frac{1,321,045}{1,321,045}(b)^7$$

$$(1.145)^{\left(\frac{1}{7}\right)} = (b^7)^{\frac{1}{7}}$$

$$1.0196 = b$$

Function – $y = 1,321,045\,(1.0196)^t$

$\downarrow$

$(1 + .0196)$

$\downarrow$

convert to %

↳ Population growing by 1.96%

population in 2013 = $y = 1,321,045\,(1.0196)^{13}$

When talking population – usually round down –

2013 population $\approx$ 1,700,220

④

9. Population in 2000 was 9426
by 2007, estimated population was 17,942.

$$y = a(b)^t$$

a $\qquad$ t $\quad$ y

$(0, 9,426) \qquad (7, 17,942)$

↑ year 2000 ↑ starting $\qquad$ ↑ year 2007 ↑ ending amt

$$\frac{17,942}{9,426} = \frac{9,426(b)^7}{9,426}$$

$$(1.9035)^{1/7} = (b^7)^{1/7}$$

$$1.0963 = b$$

Exponential Function → $y = 9426(1.0963)^t$

* population in 2012 (t) ≈ 28,410

$$y = 9426(1.0963)^{12}$$

$$y \approx 28,410$$